Mathematics — Solved Paper 2016

SECTION A (40 Marks)

*(Answer **all** questions from this Section)*

***Question* 1 :**

(a) Using remainder theorem, find the value of k if on dividing $2x^3 + 3x^2 - kx + 5$ by $x - 2$, leaves a remainder 7. [3]

Solution :

$$x - 2 = 0 \quad \Rightarrow \quad x = 2$$

Remainder = 7

$\Rightarrow$ Value of $2x^3 + 3x^2 - kx + 5$ for $x = 2$ is 7

$\Rightarrow$ $2 \times 2^3 + 3 \times 2^2 - k \times 2 + 5 = 7$

$\Rightarrow$ $16 + 12 - 2k + 5 = 7$

$\Rightarrow$ $-2k = -26$ *i.e.* $\boldsymbol{k = 13}$ **Ans.**

(b) Given A = $\begin{bmatrix} 2 & 0 \\ -1 & 7 \end{bmatrix}$ and 1 = $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ and $A^2 = 9A + mI$. Find m. [4]

Solution :

$$\mathbf{A^2 = 9A + mI} \quad \Rightarrow \quad \begin{bmatrix} 2 & 0 \\ -1 & 7 \end{bmatrix}\begin{bmatrix} 2 & 0 \\ -1 & 7 \end{bmatrix} = 9\begin{bmatrix} 2 & 0 \\ -1 & 7 \end{bmatrix} + m\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

i.e. $$\begin{bmatrix} 4 & 0 \\ -9 & 49 \end{bmatrix} = \begin{bmatrix} 18 & 0 \\ -9 & 63 \end{bmatrix} + \begin{bmatrix} m & 0 \\ 0 & m \end{bmatrix}$$

$$\Rightarrow \quad \begin{bmatrix} 4 & 0 \\ -9 & 49 \end{bmatrix} = \begin{bmatrix} 18+m & 0 \\ -9 & 63+m \end{bmatrix}$$

$\Rightarrow$ $4 = 18 + m$ *i.e.* $\boldsymbol{m = -14}$ **Ans.**

To check, take $49 = 63 + m$ *i.e.* $m = -14$

(c) The mean of following numbers is 68. Find the value of 'x'.
45, 52, 60, x, 69, 70, 26, 81 and 94.
Hence, estimate the median. [3]

Solution :

$$\frac{45+52+60+x+69+70+26+81+94}{9} = 68$$

$\Rightarrow$ $\boldsymbol{x = 115}$ **Ans.**

$\Rightarrow$ Given numbers are : 45, 52, 60, 115, 69, 70, 26, 81 and 94

Which in ascending order are : 26, 45, 52, 60, 69, 70, 81, 94 and 115

$\because$ Number of numbers, $n = 9$

$\therefore$ **Median** = $\left(\dfrac{n+1}{2}\right)^{\text{th}}$ term

$= \left(\dfrac{9+1}{2}\right)^{\text{th}}$ term = 5^{th} term = **69** **Ans.**

Question **2 :**

(a) The slope of a line joining P(6, k) and Q(1 – 3k, 3) is $\frac{1}{2}$. Find :

(i) k.

(ii) mid-point of PQ, using the value of 'k' found in (i). **[3]**

Solution :

Let P(6, k) = (x_1, y_1) and Q(1 – 3k, 3) = (x_2, y_2)

(i) Given : slope of PQ = $\frac{1}{2}$

$\Rightarrow \quad \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{1}{2} \quad$ *i.e.* $\quad \dfrac{3-k}{1-3k-6} = \dfrac{1}{2}$

$\Rightarrow \quad 6 - 2k = -3k - 5 \quad$ *i.e.* $\quad \boldsymbol{k = -11}$ **Ans.**

(ii) $k = -11 \quad \Rightarrow \quad$ P = (6, –11) and Q = (34, 3)

$\therefore$ **Mid-point of PQ** $= \left(\dfrac{x_1 + x_2}{2}, \dfrac{y_1 + y_2}{2}\right)$

$= \left(\dfrac{6+34}{2}, \dfrac{-11+3}{2}\right) = \mathbf{(20, -4)}$ **Ans.**

(b) Without using trigonometrical tables, evaluate :

$\mathrm{cosec}^2\, 57° - \tan^2 33° + \cos 44°\ \mathrm{cosec}\ 46° - \sqrt{2}\cos 45° - \tan^2 60°$ **[4]**

Solution :

Given expression $= \mathrm{cosec}^2(90° - 33°) - \tan^2 33° + \cos(90° - 46°) \cdot \mathrm{cosec}\ 46° - \sqrt{2} \times \dfrac{1}{\sqrt{2}} - \left(\sqrt{3}\right)^2$

$= \sec^2 33° - \tan^2 33° + \sin 46°\ \mathrm{cosec}\ 46° - 1 - 3$

$= 1 + \sin 46° \cdot \dfrac{1}{\sin 46°} - 4 \qquad [\because \sec^2 A - \tan^2 A = 1]$

$= 1 + 1 - 4 = \mathbf{-2}$ **Ans.**

(c) A certain number of metallic cones, each of radius 2 cm and height 3 cm, are melted and recast into a solid sphere of radius 6 cm. Find the number of cones used. **[3]**

Solution :

Let the number of cones melted = n

$\therefore$ Volume of n cones = Volume of sphere

$\Rightarrow$ $n \times$ volume of each cone = Volume of sphere

$\Rightarrow \quad n \times \dfrac{1}{3}\pi \times (2)^2 \times 3 = \dfrac{4}{3}\pi(6)^3 \quad$ *i.e.* $\quad \boldsymbol{n = 72}$ **Ans.**

Question **3 :**

(a) Solve the following inequation, write the solution set and represent it on the number line.

$-3(x - 7) \geq 15 - 7x > \dfrac{x+1}{3}$, $x \in$ R. **[3]**

Solution :

$$-3(x-7) \geq 15 - 7x > \frac{x+1}{3}$$

$\Rightarrow \quad -9x + 63 \geq 45 - 21x > x + 1$

$\Rightarrow \quad -9x + 63 \geq 45 - 21x \quad$ and $\quad 45 - 21x > x + 1$

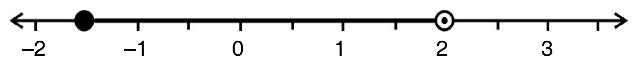
$\Rightarrow \quad 12x \geq -18 \quad$ and $\quad -22x > -44$

$\Rightarrow \quad x \geq -1{\cdot}5 \quad$ and $\quad x < 2$

$$-1{\cdot}5 \leq x < 2$$

Solution set : $\{x : x \in \text{R}$ and $-1{\cdot}5 \leq x < 2\}$

Solution on number line =

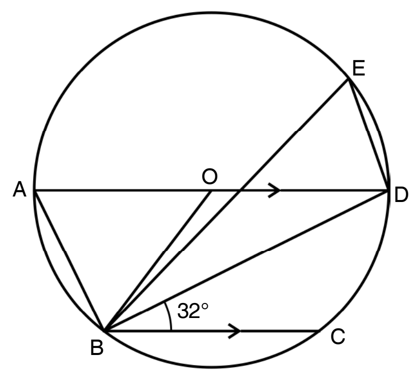


(b) In the given figure, AD is a diameter. O is the centre of the cirle. AD is parallel to BC and $\angle$CBD = 32°. Find :

(i) $\angle$OBD

(ii) $\angle$AOB

(iii) $\angle$BED **[4]**



Solution :

(i) AD // BC and BD is transversal

$\Rightarrow \quad \angle ODB = \angle DBC \quad$ [Alternate angles]

$= 32°$

Now, $\quad$ OD = OB = radius of the same circle

$\Rightarrow \quad \boldsymbol{\angle OBD} = \angle ODB$

$= \mathbf{32°}$ **Ans.**

(ii) Since, angle at centre is equal to twice the angle at remaining circumference

$\therefore \quad \boldsymbol{\angle AOB} = 2\angle ADB$

$= 2 \times 32° = \mathbf{64°}$ **Ans.**

(iii) $\quad \angle AOB + \angle BOD = 180° \quad$ [AD is a straight line]

$\Rightarrow \quad 64° + \angle BOD = 180°$

$\Rightarrow \quad \angle BOD = 116°$

Also, $\quad \angle BOD = 2\angle BED$

$\Rightarrow \quad 116° = 2 \times \angle BED$ *i.e.* $\boldsymbol{\angle BED = 58°}$ **Ans.**

(c) If (3*a* + 2*b*) : (5*a* + 3*b*) = 18 : 29. Find *a* : *b*. **[3]**

Solution :

$(3a + 2b) : (5a + 3b) = 18 : 29$

$\Rightarrow \quad \dfrac{3a+2b}{5a+3b} = \dfrac{18}{29} \quad$ *i.e.* $\quad 87a + 58b = 90a + 54b$

$\Rightarrow \quad 3a = 4b \quad$ *i.e.* $\quad \dfrac{a}{b} = \dfrac{4}{3} \quad$ *i.e.* $\boldsymbol{a : b = 4 : 3}$ **Ans.**

Question 4 :

(a) A game of numbers has cards marked with 11, 12, 13, ..., 40. A card is drawn at random. Find the probability that the number on the card drawn is :

(i) A perfect square. **(ii) Divisible by 7.** **[3]**

Solution :

Total number of outcomes = 30 (11, 12, 13, ..., 40)

(i) Perfect square number are : 16, 25 and 36 *i.e.* 3.

∴ **Probability that the number on the card drawn is a perfect square**

$$= \frac{3}{30} = \mathbf{\frac{1}{10}}$$ **Ans.**

(ii) **Numbers divisible by 7** are 14, 21, 28 and 35 *i.e.* 4.

∴ **Required probability** $= \frac{4}{30} = \mathbf{\frac{2}{15}}$. **Ans.**

(b) Use graph paper for this question. **[3]**

(Take 2 cm = 1 unit along both *x*-axis and *y*-axis.)

Plot the points O(0, 0), A(–4, 4), B(–3, 0) and C(0, –3)

(i) Reflect points A and B on the *y*-axis and name them A′ and B′ respectively. Write down their co-ordinates. **[4]**

(ii) Name the figure OABCB′A′.

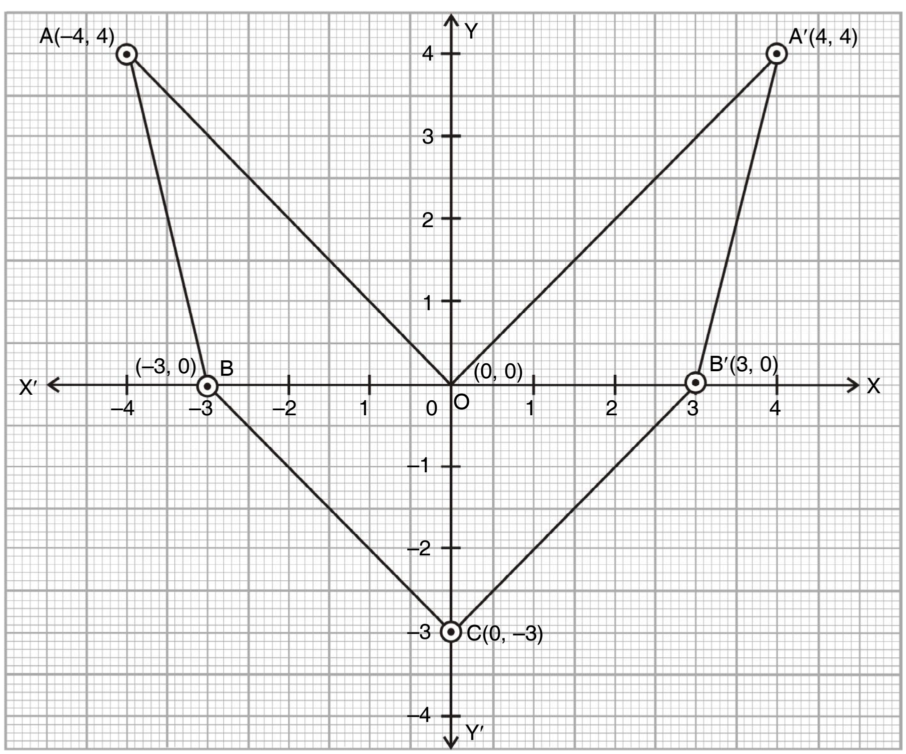
(iii) State the line of symmetry of this figure. [Now symmetry is not in course]

Solution :

(i) **A′ = (4, 4)** and **B′ = (3, 0)** **Ans.**

(ii) **Similar to an arrow-head.** **Ans.**

(iii) ***y*-axis *i.e.* *x* = 0.** [Now symmetry is not in course] **Ans.**



(c) Mr. Lalit invested ₹ 5,000 at a certain rate of interest, compounded annually for two years. At the end of first year it amounts to ₹ 5,325. Calculate :

(i) The rate of interest.

(ii) The amount at the end of second year, to the nearest rupee. [3]

[This question is not in ICSE now]

SECTION B (40 Marks)

*(Answer any **four** questions from this Section)*

***Question* 5 :**

(a) Solve the quadratic equation $x^2 - 3(x + 3) = 0$; Give your answer correct to two significant figures. [3]

Solution :

$x^2 - 3(x + 3) \Rightarrow x^2 - 3x - 9 = 0$

Comparing with $ax^2 + bx + c = 0$, we get :

$a = 1$, $b = -3$ and $c = -9$

$$b^2 - 4ac = (-3)^2 - 4 \times 1 \times -9$$

$$= 9 + 36 = 45$$

$$\sqrt{b^2 - 4ac} = \sqrt{45} = 6{\cdot}708$$

$$\therefore \quad \boldsymbol{x} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{3 \pm 6{\cdot}708}{2 \times 1} = \frac{3 + 6{\cdot}708}{2} \text{ or } \frac{3 - 6{\cdot}708}{2}$$

$= 4{\cdot}854$ or $-1{\cdot}854$ = **4·9** or **–1·9** **Ans.**

(b) A page from the savings bank account of Mrs. Ravi is given below :

Date	Particulars	Withdrawal (₹)	Deposit (₹)	Balance (₹)
April 3rd, 2006	B/F	–	–	6,000
April 7th	By cash	–	2,300	8,300
April 15th	By cheque	–	3,500	11,800
May 20th	To self	4,200	–	7,600
June 10th	By cash	–	5,800	13,400
June 15th	To self	3,100	–	10,300
August 13th	By cheque	–	1,000	11,300
August 25th	To self	7,400	–	3,900
Sept. 6th, 2006	By cash	–	2,000	5,900

She closed the account on 30th September, 2006. Calculate the interest Mrs. Ravi earned at the end of 30th September, 2006 at 4·5% per annum interest. Hence, find the amount she receives on closing the account. [4]

[Not in ICSE now]

(c) In what time will ₹ 1,500 yield ₹ 1,996.50 as compound interest at 10% per annum compounded annually ? [3]

[Not in ICSE now]

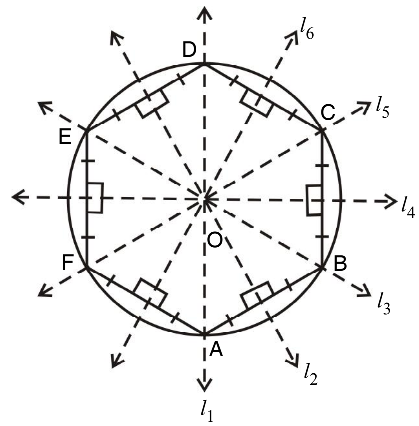
Question **6 :**

(a) Construct a regular hexagon of side 5 cm. Hence, construct all its lines of symmetry and name them. [3]

[Lines of symmetry is not in the course now]

Solution :

Step 1 : With any point O as centre, draw a circle of radius 5 cm (equal to the length of the side of the required hexagon).



Step 2 : Taking any point A, on the circumference of the circle drawn, as centre and radius 5 cm (side of the hexagon), draw an arc which cuts the circle drawn at point B.

Step 3 : Draw arcs BC = CD = DE = EF = AB, each of radius 5 cm and intersecting the circle drawn at points C, D, E and F.

Step 4 : Join AB, BC, CD, DE, EF and FA to get the regular hexagon of each side 5 cm.

Step 5 : Draw lines alongs the diameters AD, BE and CF. Also, draw perpendicular bisectors of the sides of the regular hexagon.

We shall be getting three lines along the diagonals and three lines as the perpendicular bisectors of the sides.

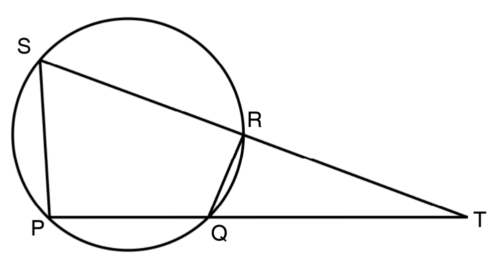
Thus, 3 lines + 3 lines = 6 lines obtained are the required lines of symmetry, which are shown by dotted lines.

(b) In the given figure, PQRS is a cyclic quadrilateral. PQ and SR produced meet at T.

(i) Prove ΔTPS ~ ΔTRQ.

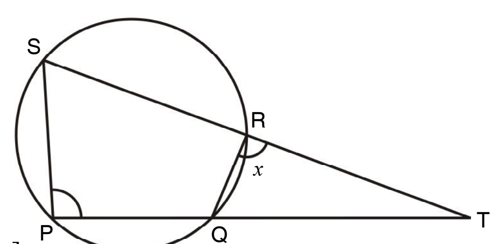
(ii) Find SP, if TP = 18 cm, RQ = 4 cm and TR = 6 cm.

(iii) Find area of quadrilateral PQRS, if area of ΔPTS = 27 cm^2. [4]



Solution :

(i) Since, in a cyclic quadrilateral, exterior angle is equal to its interior opposite angle



$\therefore$ $\angle x = \angle P$

$\angle T = \angle T$ [Common]

$\therefore$ **ΔTPS ~ ΔTRQ** [By A.A. axiom]

(ii) $\Delta TPS \sim \Delta TRQ$

$$\Rightarrow \quad \frac{SP}{RQ} = \frac{TP}{TR} \quad \textit{i.e.} \quad \frac{SP}{4\,\text{cm}} = \frac{18\,\text{cm}}{6\,\text{cm}}$$

$\Rightarrow$ **SP = 12 cm** **Ans.**

(iii) $\Delta TPS \sim \Delta TRQ$

$$\Rightarrow \quad \frac{\text{Ar. TPS}}{\text{Ar. TRQ}} = \frac{SP^2}{RQ^2} = \frac{12^2}{4^2} = \frac{144}{16} = \frac{9}{1}$$

$$\Rightarrow \quad \frac{\text{Ar. } \Delta TPS}{\text{Ar. } \Delta TPS - \text{Ar. } \Delta TRQ} = \frac{9}{9-1}$$

$$\Rightarrow \quad \frac{27\text{ cm}^2}{\text{Ar. of quad. PQRS}} = \frac{9}{8}$$ *i.e.* **Ar. (quad. PQRS) = 24 cm²** **Ans.**

(c) Given matrix A = $\begin{bmatrix} 4\sin 30^\circ & \cos 0^\circ \\ \cos 0^\circ & 4\sin 30^\circ \end{bmatrix}$ and B = $\begin{bmatrix} 4 \\ 5 \end{bmatrix}$. If AX = B.

(i) Write the order of matrix X.

(ii) Find the matrix 'X'. **[3]**

Solution :

$$\textbf{AX = B} \Rightarrow \begin{bmatrix} 4\sin 30^\circ & \cos 0^\circ \\ \cos 0^\circ & 4\sin 30^\circ \end{bmatrix} \cdot X = \begin{bmatrix} 4 \\ 5 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} \cdot X = \begin{bmatrix} 4 \\ 5 \end{bmatrix} \qquad \left[\because \sin 30^\circ = \frac{1}{2} \text{ and } \cos 0^\circ = 1\right]$$

(i) Let order of matrix X $= a \times b$

$$\therefore \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}_{2\times 2} \cdot X_{a \times b} = \begin{bmatrix} 4 \\ 5 \end{bmatrix}$$

$\Rightarrow$ $a = 2$ and $b = 1$

$\therefore$ **Order of matrix X** $= a \times b =$ **2 × 1** **Ans.**

(ii) Let matrix X $= \begin{bmatrix} x \\ y \end{bmatrix}$

$$\therefore \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 4 \\ 5 \end{bmatrix}$$

$\Rightarrow$ $2x + y = 4$ and $x + 2y = 5$

$\Rightarrow$ $x = 1$ and $y = 2$

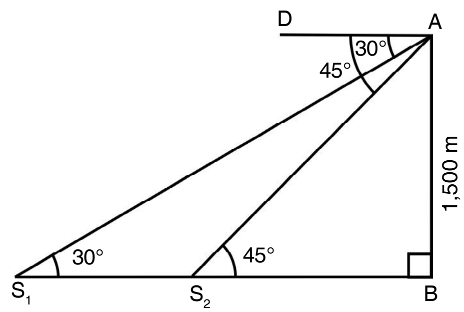
$\therefore$ **Matrix X** $= \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \mathbf{1} \\ \mathbf{2} \end{bmatrix}$ **Ans.**

Question 7 :

(a) An aeroplane, at an altitude of 1,500 metres, finds that two ships are sailing towards it in the same direction. The angles of depression as observed from the aeroplane are 45° and 30° respectively. Find the distance between the two ships. [4]

Solution :

Let position of the aeroplane be at point A, AB is perpendicular to the horizontal surface through the two ships. S_1 and S_2 be the positions of the two ships such that $\angle S_1AD = \angle AS_1B = 30°$ and $\angle S_2AD = \angle AS_2B = 45°$.



Clearly, we are to find the distance between S_1 and S_2 *i.e.* S_1S_2.

It is given that the aeroplane is at an altitude of 1500 m *i.e.* AB = 1500 m.

In Δ ABS_2, $\tan 45° = \dfrac{AB}{BS_2}$

i.e. $1 = \dfrac{1500\,\text{m}}{BS_2} \Rightarrow BS_2 = 1500$ m

In Δ ABS_1, $\tan 30° = \dfrac{AB}{BS_1}$

i.e. $\dfrac{1}{\sqrt{3}} = \dfrac{1500\,\text{m}}{BS_1} \Rightarrow BS_1 = 1500 \times \sqrt{3}$ m = 2598 m

$\therefore$ **Distance between the two ships** $= BS_1 - BS_2$

= 2598 m – 1500 m = **1098 m** **Ans.**

(b) The table shows the distribution of the scores obtained by 160 shooters in a shooting competition. Use a graph sheet and draw an ogive for the distribution. (Take 2 cm = 10 scores on the X-axis and 2 cm = 20 shooters on the Y-axis.)

Scores	**0-10**	**10-20**	**20-30**	**30-40**	**40-50**	**50-60**	**60-70**	**70-80**	**80-90**	**90-100**
No. of shooters	**9**	**13**	**20**	**26**	**30**	**22**	**15**	**10**	**8**	**7**

Use your graph to estimate the following :

(i) The median.

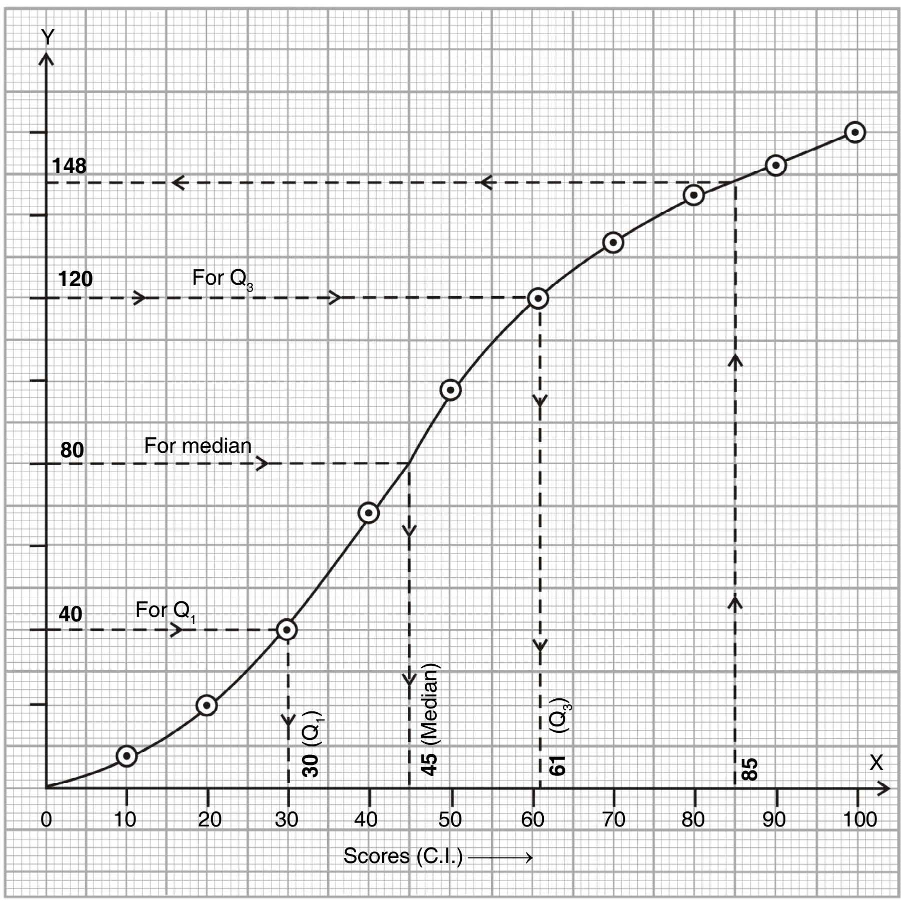
(ii) The interquartile range.

(iii) The number of shooters who obtained a score of more than 85%. [6]

Solution :

Scores C.I.	No. of shooters (*f*)	*c.f.*
0-10	9	9
10-20	13	22
20-30	20	42
30-40	26	68
40-50	30	98
50-60	22	120
60-70	15	135
70-80	10	145
80-90	8	153
90-100	7	**160**
	160	

With the given data, the ogive drawn will be as shown below :



(i) **The median** $= \left(\dfrac{160}{2}\right)^{\text{th}}$ term

$= 80^{\text{th}}$ term $=$ **45** **Ans.**

(ii) $\because$ Upper quartile (Q_3) $= \left(\frac{3}{4}\times 160\right)^{th}$ term

$= 120^{th}$ term $= 61$ (app.)

and, lower quartile (Q_1) $= \left(\frac{1}{4}\times 160\right)^{th}$ term

$= 40^{th}$ term $= 30$ (app.)

$\therefore$ **The interquartile range** $= Q_3 - Q_1$

$= 61 - 30 =$ **31 (app.)** **Ans.**

(iii) $\because$ 85% of 100 $= 85$

And, at score $= 85$, shooter's number $= 148$

$\therefore$ The number of shooters who obtained a score of more than 85%

$= 160 - 148 =$ **12** **Ans.**

***Question* 8 :**

(a) If $\frac{x}{a} = \frac{y}{b} = \frac{z}{c}$ show that : $\frac{x^3}{a^3} + \frac{y^3}{b^3} + \frac{z^3}{c^3} = \frac{3xyz}{abc}$. **[3]**

Solution :

Let $\frac{x}{a} = \frac{y}{b} = \frac{z}{c} = k$

$\Rightarrow$ $x = ak$, $y = bk$ and $z = ck$

$\therefore$ **L.H.S.** $= \frac{x^3}{a^3} + \frac{y^3}{b^3} + \frac{z^3}{c^3}$

$= \frac{a^3k^3}{a^3} + \frac{b^3k^3}{b^3} + \frac{c^3k^3}{c^3}$

$= k^3 + k^3 + k^3$

$= 3k^3 = 3kkk = 3\cdot\frac{x}{a}\cdot\frac{y}{b}\cdot\frac{z}{c} = \frac{3xyz}{abc}$ = **R.H.S.**

(b) Draw a line AB = 5 cm. Mark a point C on AB such that AC = 3 cm. Using a ruler and a compass only, construct :

(i) A circle of radius 2·5 cm, passing through A and C.

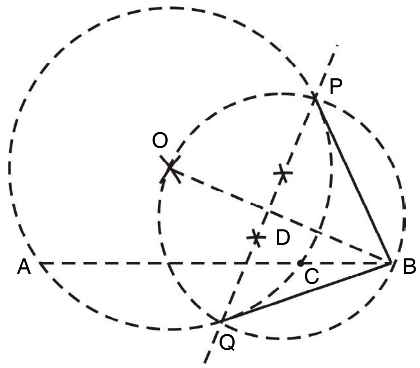
(ii) Construct two tangents to the circle from the external point B. Measure and record the length of the tangents. **[4]**

Solution :

Steps :

1. Draw line AB = 5 cm.
2. From AB cut AC = 3 cm.

3. With A and C as centres and with radius 2·5 cm in each case, draw two arcs intersecting each other at point O.
4. With O as centre and radius equal to 2·5 cm, draw a circle that will pass through the points A and C.
5. Join O and B.
6. Draw perpendicular bisector of OB that meets OB at point D.



7. With point D as centre and OD = BD as radius, draw a circle that will intersect the first circle at point P and Q.
8. Join PB and QB.

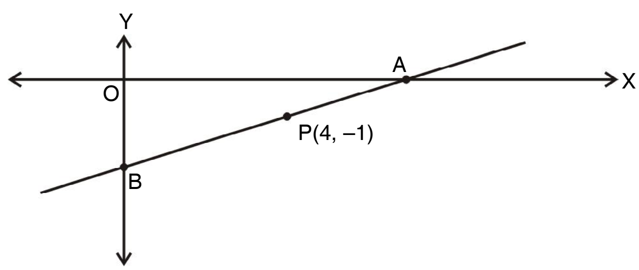
PB and QB are the required two tangents.

Each of tangents PB and QB = 3·2 cm (approx) **Ans.**

(c) A line AB meets X-axis at A and Y-axis at B. P(4, –1) divides AB in the ratio 1 : 2.

(i) Find the co-ordinates of A and B.

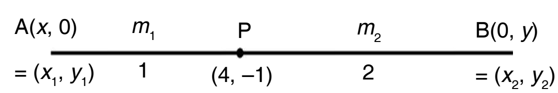
(ii) Find the equation of the line through P and perpendicular to AB. **[3]**



Solution :

(i) Let A = $(x, 0)$ and B = $(0, y)$

$$P(x) = \frac{m_1x_2 + m_2x_1}{m_1 + m_2}$$



$$\Rightarrow \quad 4 = \frac{1 \times 0 + 2 \times x_1}{1 + 2}$$

$$\Rightarrow \quad 12 = 2x_1 \quad i.e. \quad x_1 = 6$$

$$P(y) = \frac{m_1y_2 + m_2y_1}{m_1 + m_2} \Rightarrow -1 = \frac{1 \times y_2 + 2 \times 0}{1 + 2} \quad i.e. \quad y_2 = -3$$

$\therefore$ **A** $= (x, 0) = (x_1, 0) =$ **(6, 0)**

and **B** $= (0, y) = (0, y_2) =$ **(0, –3)** **Ans.**

(ii) $\because$ Slope of AB $= \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{-3-0}{0-6} = \dfrac{1}{2}$

$\therefore$ Slope of perpendiuclar to AB = –2

Required equation is

$$
\begin{aligned}
y - y_1 &= m(x - x_1) \\
\Rightarrow \quad y + 1 &= -2(x - 4) \\
\Rightarrow \quad y + 1 &= -2x + 8 \\
\textit{i.e.} \quad \mathbf{2x + y} &= \mathbf{7}
\end{aligned}
$$

Taking, slope $m = -2$ and, $(x_1, y_1) = P(4, -1)$

Ans.

***Question* 9 :**

(a) A dealer buys an article at a discount of 30% from the wholesaler, the marked price being ₹ 6,000. The dealer sells it to a shopkeeper at a discount of 10% on the marked price. If the rate of VAT is 6%, find :

(i) The price paid by the shopkeeper including the tax.

(ii) The VAT paid by the dealer. [3]

Solution :

Given marked price (M.P.) = ₹ 6,000

For dealer :

C.P. = ₹ 6,000 – 30% of ₹ 6,000

$= ₹\ 6{,}000 - \dfrac{30}{100} \times ₹\ 6{,}000 = ₹\ 4{,}200$

S.P. = ₹ 6,000 – 10% of ₹ 6,000

$= ₹\ 6{,}000 - \dfrac{10}{100} \times ₹\ 6{,}000 = ₹\ 5{,}400$

(i) **For shopkeeper :**

C.P. = ₹ 5,400

$\Rightarrow$ Price paid = ₹ 5,400 + 6% of ₹ 5,400

= **₹ 5,724**

(ii) **VAT paid by the dealer**

= Tax on S.P. – Tax on C.P.

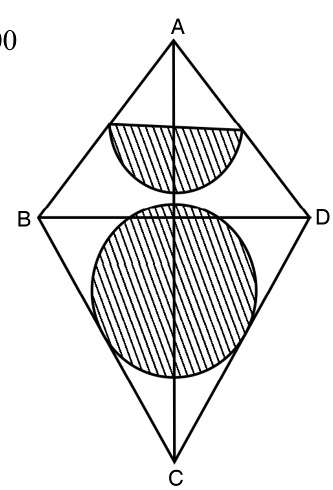
= 6% of ₹ 5,400 – 6% of ₹ 4,200

= 6% of ₹ 1,200 = **₹ 72** **Ans.**

(b) The given figure represents a kite with a circular and a semicircular motifs stuck on it. The radius of circle is 2·5 cm and the semicircle is 2 cm. If diagonals AC and BD are of lengths 12 cm and 8 cm respecticely, find the area of the :

(i) shaded part. Give your answer correct to the nearest whole number.

(ii) unshaded part. [4]



Solution :

(i) **Area of shaded part**

$$= \pi \times (2\cdot5)^2 \text{ cm}^2 + \frac{1}{2} \times \pi \times (2)^2 \text{ cm}^2$$

$$= \frac{22}{7} \times (6\cdot25 + 2) \text{ cm}^2$$

$$= \frac{22}{7} \times 8\cdot25 \text{ cm}^2 = 25\cdot928 \text{ cm}^2 = \mathbf{26 \text{ cm}^2}$$ **Ans.**

(ii) $\because$ Area of kite ABCD $= \frac{1}{2} \times AC \times BD$

$$= \frac{1}{2} \times 12 \times 8 \text{ cm}^2 = 48 \text{ cm}^2$$

$\therefore$ **Area of unshaded part** $= 48 \text{ cm}^2 - 26 \text{ cm}^2$

$$= \mathbf{22 \text{ cm}^2}$$ **Ans.**

(c) A model of a ship is made to a scale 1 : 300.

(i) The length of the model of the ship is 2 m. Calculate the length of the ship.

(ii) The area of the deck of the ship is 180,000 m^2. Calculate the area of the deck of the model.

(iii) The volume of the model is 6·5 m^3. Calculate the volume of the ship. [3]

Solution :

Given scale factor $k = \frac{1}{300}$

(i) $\because$ The length of the model of the ship

$= k \times$ the length of the ship

$\Rightarrow$ $2 \text{ m} = \frac{1}{300} \times$ the length of the ship

$\Rightarrow$ **The length of the ship = 600 m** **Ans.**

(ii) **Area of the deck of model** $= k^2 \times$ area of the deck of the ship

$$= \left(\frac{1}{300}\right)^2 \times 180{,}000 \text{ m}^2 = \mathbf{2 \text{ m}^2}$$ **Ans.**

(iii) $\because$ Volume of the model $= k^3 \times$ volume of the ship

$\Rightarrow$ $6\cdot5 \text{ m}^3 = \left(\frac{1}{300}\right)^3 \times$ volume of the ship

$\Rightarrow$ **Volume of the ship** $= 6\cdot5 \times 300 \times 300 \times 300 \text{ m}^3$

$$= \mathbf{17{,}55{,}00{,}000 \text{ m}^3}$$ **Ans.**

***Question* 10 :**

(a) Mohan has a recurring deposit account in a bank for 2 years at 6% p.a. simple interest. If he gets ₹ 1,200 as interest at the time of maturity, find :

(i) the monthly instalment

(ii) the amount of maturity. [3]

Solution :

Given time, n = 2 years = 24 months; Rate, r = 6% and interest, I = ₹ 1,200

(i) Required to find monthly instalment = P

$$\because \qquad I = P \times \frac{n(n+1)}{2 \times 12} \times \frac{r}{100}$$

$$\Rightarrow \qquad 1{,}200 = P \times \frac{24 \times 25}{2 \times 12} \times \frac{6}{100} \quad \Rightarrow \quad P = \frac{1{,}200 \times 24 \times 100}{24 \times 25 \times 6} = 800$$

$\therefore$ **Monthly instalment = ₹ 800** **Ans.**

(ii) $\because$ Sum deposited = P × n

= ₹ 800 × 24 = ₹ 19,200

$\therefore$ **The amount of maturity** = Sum deposited + Interest on it

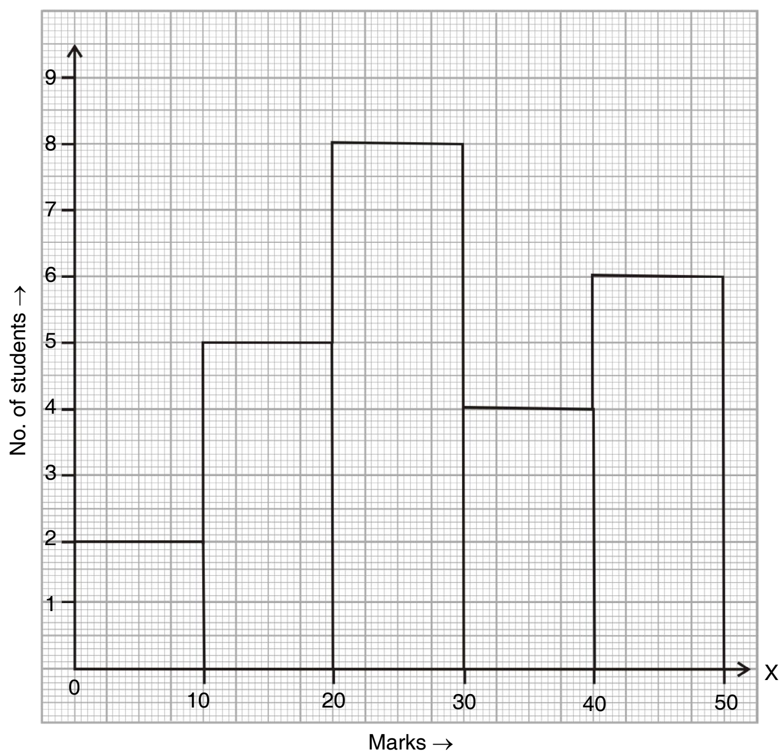
= ₹ 19,200 + ₹ 1,200

= **₹ 20,400** **Ans.**

(b) The histogram below represents the scores obtained by 25 students in a Mathematics mental test. Use the data to :

(i) Frame a frequency distribution table. **(ii) To calculate mean.**

(iii) To determine the Modal class. **[4]**



Solution :

(i) **Required frequency distribution table is :**

C.I.	**0-10**	**10-20**	**20-30**	**30-40**	**40-50**
Frequency	**2**	**5**	**8**	**4**	**6**

Ans.

(ii)

C.I.	f	Class-mark (x)	$f \times x$
0-10	2	5	10
10-20	5	15	75
20-30	8	25	200
30-40	4	35	140
40-50	6	45	270
	$\Sigma f = 25$		$\Sigma f \times x = 695$

$\therefore$ **Mean** $= \dfrac{\sum f \times x}{\sum f} = \dfrac{695}{25} = \mathbf{27{\cdot}8}$ **Ans.**

(ii) **Modal class** = Class with maximum frequency

= **20 – 30** **Ans.**

(c) A bus covers a distance of 240 km at a uniform speed. Due to heavy rain its speed gets reduced by 10 km/h and as such it takes two hrs longer to cover the total distance. Assuming the uniform speed to be '*x*' km/h, form an equation and solve it to evaluate '*x*'. **[3]**

Solution :

In 1st case :

$$\text{time} = \frac{\text{distance}}{\text{speed}} = \frac{240}{x} \text{ hrs}$$

In 2nd case (during the heavy rain) **:**

$$\text{speed} = (x - 10) \text{ km/h}$$

and, $\text{time} = \dfrac{\text{distance}}{\text{speed}} = \dfrac{240}{x-10}$ hrs

Given : $\dfrac{240}{x-10} - \dfrac{240}{x} = 2$

i.e. $\dfrac{240x - 240x + 2400}{x(x-10)} = 2$

i.e. $2x^2 - 20x = 2400 \Rightarrow x^2 - 10x - 1200 = 0$

i.e. $x^2 - 40x + 30x - 1200 = 0 \Rightarrow x(x - 40) + 30(x - 40) = 0$

i.e. $(x - 40)(x + 30) = 0 \Rightarrow x = 40$ or $x = -30$

i.e. $\mathbf{x = 40}$ **Ans.**

***Question* 11 :**

(a) Prove that : $\mathbf{\dfrac{\cos A}{1 + \sin A} + \tan A = \sec A.}$ **[3]**

Solution :

L.H.S. $= \dfrac{\cos A}{1 + \sin A} + \dfrac{\sin A}{\cos A}$

$$= \frac{\cos^2 A + \sin A + \sin^2 A}{\cos A(1+\sin A)}$$

$$= \frac{1+\sin A}{\cos A(1+\sin A)} \qquad [\because \cos^2 A + \sin^2 A = 1]$$

$$= \frac{1}{\cos A} = \sec A = \textbf{R.H.S.}$$ **Hence Proved.**

(b) Use ruler and compasses only for the following question. All construction lines and arcs must be clearly shown.

(i) Construct a ΔABC in which BC = 6·5 cm, ∠ABC = 60°, AB = 5 cm.

(ii) Construct the locus of points at a distance of 3·5 cm from A.

(iii) Construct the locus of points equidistant from AC and BC.

(iv) Mark 2 points X and Y which are at a distance of 3·5 cm from A and also equidistant from AC and BC. Measure XY. **[4]**

Solution :

(i) **Steps :**

Draw BC = 6·5 cm. Draw line PB so that ∠PBC = 60°. From PB, cut BA = 5 cm

ΔABC is the required triangle. **Ans.**

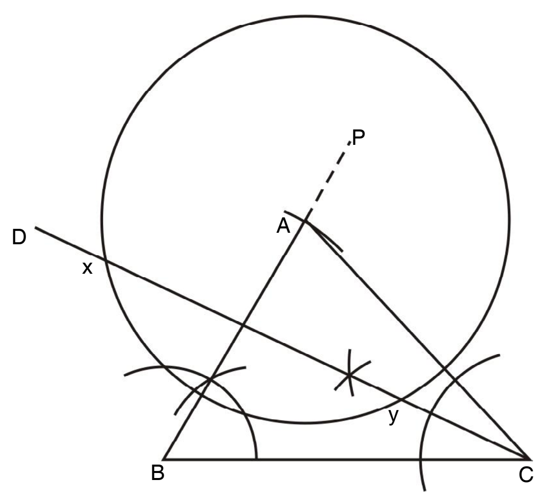
(ii) Draw a circle with centre at A and radius = 3·5 cm.

Circumference of the circle is the required locus. **Ans.**

(iii) Draw CD, which is bisector of angle ACB.

CD is the required locus. **Ans.**

(iv) Circle with centre A and line CD meet at points X and Y. Which are shown in the figure drawn. **XY** = **8·2 cm** (approximately) **Ans.**



(c) Ashok invested ₹ 26,400 in 12%, ₹ 25 shares of a company. If he receives a dividend of ₹ 2,475, find the :

(i) Number of shares he bought.

(ii) Market value of each share. **[3]**

Solution :

(i) Total dividend = ₹ 2,475

and, dividend on each share = 12% of ₹ 25

$$= ₹\ \frac{12}{100} \times 25 = ₹\ 3$$

$\therefore$ **Number of shares bought** $= \dfrac{\text{Total dividend}}{\text{Dividend on 1 share}}$

$$= \frac{₹\ 2{,}475}{₹\ 3} = \mathbf{825}$$ **Ans.**

(ii) Market value of all the 825 shares = ₹ 26,400

$\Rightarrow$ **Market value of each share** $= \dfrac{₹\ 26{,}400}{825} = $ **₹ 32** **Ans.**